Exam. Code : 211004
Subject Code : 4290

M.Sc. (Mathematics) 4th Semester

DISCRETE MATHEMATICS–I

Paper—MATH–575

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Candidates are required to attempt **TWO** questions from each Unit. Each question carries equal marks.

UNIT—I

1. (a) Prove that if any 14 numbers from 1 to 25 are chosen then one of them is a multiple of another.

   (b) Let T be an equilateral triangle whose sides are of length 1 unit. Show that if any five points are chosen lying on or inside triangle, then two of them must be no more than 1/2 unit apart.

2. (a) In a group of 70 people, 37 likes coffee, 52 like tea and each person likes at least one of two drinks. How many like both coffee and tea ?

   (b) A and B are two sets that $n(A-B) = 15+x$, $n(B-A) = 3x$ and $n(A \cap B) = x$. Find the value of x and $n(A \cup B)$.

3. Explain equivalence relation and partial order relation with examples. What is difference between them ?

4. (a) Give an example of a relation which is anti symmetric and transitive, but neither reflexive nor symmetric.

   (b) Prove that a function $f : A \rightarrow A$ has an inverse if and only if f is one to one and onto.

UNIT—II

5. Explain different kinds of Statements with examples.

6. (a) State and prove Detachment condition of logic.

   (b) State and prove De Morgan law.

7. (a) Prove that the conditional operation distributes over the operation of conjunction.

   (b) Write the converse, inverse and negation of the following statement :

   "If Sandra finishes her work, she will go to the basket ball game unless it snows"

8. Explain various laws of logic with examples.

UNIT—III

9. (a) Show that the intersection of two congruence relation on a semi group is a congruence.

   (b) Let S = {a,b}, write the operation table for semi group S^S. Is the semi group commutative ?

10. State and prove Fundamental Theorem of semi group homomorphism.

11. Prove that G/H is a sub group where G is a group and H is a collection of all right cosets in G.

12. (a) Explain congruence relation with examples on semi group and monoids.

    (b) Prove that a finite semi group in which cancellation laws hold is a group.

UNIT—IV

13. (a) Write a short note on Phase structure grammar.

    (b) What is meaning of regular in grammars ? Give example of it.

14. Explain context free and context sensitive languages with examples. Write two applications of these.

15. Explain derivation sentential forms of grammars and language.

16. Explain various writing rules of a grammars with examples.

UNIT—V

17. If $S_n - 6S_{n-1} + 5S_{n-2} = 0$, $S_0 = 2$, $S_1 = 2$, find generating function :

    (i) Using definition of generating function

    (ii) Using operation on sequences and their generating functions.

    (iii) Write solution of recurrence relation.

18. Solve $S_{n+2} - S_{n+1} - S_n = 0$ if $S_0 = 1$, $S_1 = 1$.

19. Explain various types of relations with examples.

20. (a) State and prove recursion theorem.

    (b) Explain various standard sequence of generating functions.